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Dedicated to
Elena Sassi

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Geometric Historical Approach to Investigate Celestial Bodies with a full Digital Planetarium

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Abstract

The educational values of activities with Digital Planetarium are increasingly acknowledged. Since 2009, the Digital Planetarium of Caserta (PdC, http://www.planetariodicaserta.it), built with support by City Municipality and EU funds Urban2, develops original activities aiming at increasing scientific competencies of students (Primary ←→ University), teachers and citizens according to active-learning strategies. At PdC the topics are discussed in an advanced multi-media ambient (the full-dome) in real-time with the help of a professional scientific tutor. Participants live an intense cognitive experience because of innovative technology and methods used. Before the PdC activities students are involved in lectures, web-search and individual study to build a basic background. Later, the didactic path continues in class by discussing observations, physical and mathematical features, critical aspects of historical methods, their success or failure and the approximations involved. In 2009 - 2012, at secondary school level, about 240 schools, 350 teachers and 11000 students have been involved. The here described "Investigating the near Space by means of Geometry" activity proposes several celebrate astronomical experiences since the Ancient Age (Eratosthenes, Aristarchus) up to Galileo (1610) and Halley (1716). Geometrical Euclidean methods are used to estimate dimensions and distances in Solar System (Earth radius, angular dimension of Sun and Moon, Moon diameter, Moon and Sun distances to the Earth, size of Moon's mountains and distance of closer stars). For these experiences, the activity presents on the dome historical contexts, main astronomical properties of the celestial bodies involved; the original measurements are performed and compared with mathematical models. The Educational Added Value (EAV) of the activity has been tested with 42 students (17-19 years old), characterised as "expert" (extra training in Physics and Mathematics school activities) and "non-expert", via a 45 minutes PdC session and pre-post questionnaires. The results indicate an increased specific knowledge.

Keywords: Physics and Astronomy, full digital planetarium, geometrical historical approach, students/ teachers education, educational validation procedures.

Introduction

The educational activities based on working with reconstructions of the sky and celestial bodies, from catalogues of stars, pictures of planets, etc…, are becoming more and more valued (Hobson, Trubdle, & Saçkes, 2010). The analysis of these activities is useful also to reflect on the Educational Added Value (EAV) that such experiences can add to formal, non-formal and informal education at different age levels. The dynamical images (3D objects) built on the dome of a Digital Planetarium (DP) are not simulations but reconstructions from real data in an advanced multi-media environment; this very fact usually increases the motivation of the participants and helps to build or deepen their scientific knowledge. Nowadays high quality astronomical images of celestial bodies are on the web; the EAV of activities in a fully DP is much enhanced by the fact that the representation on the dome is dynamical and comes from acknowledged large databases of measured quantities. Well designed and implemented activities propose to the participant experiences that are cognitively dense because of the innovative technology as well as the methodological approaches used.

The difficulty of understanding astronomy is due in part to concepts involving geometries and orientations of celestial bodies in three dimensions. Students have to build conceptual knowledge about a three-dimensional (3D) physical space while being taught using two-dimensional (2D) textbook materials (Ku, 2005). Evaluations of student learning in an immersive full-dome digital theatre indicate that the immersive experiences created by full-dome video enhance learning – especially of difficult concepts requiring students to change reference frames (Sumnersa, Reiffb, & Weberc, 2008). The great potential of full-dome in teaching and learning involves relevant psychological aspects (visual perception, attention, memory, social factors and individual differences) that needs to be investigated (Schnalla, Hedgeb, & Weaverc, 2012).

In this paper the activity "Investigating the near Space by means of Geometry", developed at the Digital Planetarium of Caserta (PdC)[1] (http://www.planetariodicaserta.it) in 2012, by L.A. Smaldone and P. Di Lorenzo, is described as well as the reaction of forty-two students (16 -19 years old) attending a secondary school.

Two research hypotheses are taken into account in the qualitative research described in the following: a) integration of Digital Planetarium (DP) activities in the syllabus of an ordinary secondary school; b) contribution of a specific DP activity to the improvement of historical, astronomical and geometrical knowledge of secondary school students.

The PdC develops original activities devoted to increase scientific capabilities in students, in-service teachers, and general public.

Up to Fall 2012, twenty-tree original activities have been developed and offered; each of them is appropriate to a specific target/public, according to different level of abilities, age, knowledge, etc. The general objectives of each activity can be summarised as: - to improve the basic scientific knowledge of students and teachers through experiences based on digital representation of celestial phenomena; - to link with salient achievements in History of Physics and Astronomy; - to help in-service teachers[2] develop deeper competences on the addressed topics; - to offer high-quality scientific edutainment to citizens of different age. The realization of an activity requires a synergic combination of different expertises and capabilities (storyboard, construction of 3D object and images from real data, relevant images selection, music selection, programming, preparation of pre/post class-work, etc. ...). On average more than two months of work are needed to build a forty-five minutes full-dome real-time activity.

The Italian school system is centralised, i.e. curricula and syllabuses are defined by the Ministry of Education. The vast majority of schools are State schools (e.g. 90 out of 117 schools in the Caserta Municipality area) and the teachers are State employees. Schematically: Primary school (5 years, students' age about 6 -10); First Grade Secondary School (3 years, age about 11 -13); Second Grade Secondary School (5 years, age about 14 -18). University education is organized according to the Bologna schema (Bachelor, Master, Ph.D.)

As far as the Primary and Secondary school level is concerned, in 2009 – 2012 about 240 schools, 350 teachers and 11000 students have been involved in the PdC activities. This impact is significant given the about 1400 schools of the Caserta Province. Many full-dome real-time activities have been up to now developed, e.g. Earth motion, Moon characteristic, Solar System, Galileo's findings, Kepler laws, stellar evolution, orienting by means of star positions and motions, etc…; seven for the Primary school teachers and students, sixteen for the Secondary level. In all activities for the schools, active-learning strategies and approaches are used to facilitate teachers' and students' education (for a review on active learning in astronomy see Prather, Rudolph, & Brissend, 2009). Different levels of depth are proposed to diverse age levels.

Usually the fruition of a PdC activity is not a once-in-a-while event but part of a didactical path articulated in three phases: a) preparation in class through presentations by teachers, students' study on books and via web-search, etc… to build a basic background for the topic to be addressed and to clarify the learning goals; b) the activity at PdC focused on representing the topic on the dome, discussing Astronomy, Physics, Mathematics aspects (almost always from different viewpoints, e.g. Earth vs Space), solving problems and using models; c) continuation in class to expand the content addressed at the Planetarium, to assess the understanding, etc…

1 The Digital Planetarium of Caserta (PdC), built in 2009 with the support of the City Municipality and EU funds Urban2, is based on *In Space System*, a cluster of 7 PC, 5 DLP projectors, Dolby surround 5.1, in a dome (diameter = 7 m) overlooking 42 seats. *SkyExplorer* allows developing the scientific / astronomical objectives in an object-oriented programming language. *In Space System* and *SkyExplorer* are trademarks of RSACosmos.

2 Currently a program for teachers-to-be is being developed.

In the following an example activity is described, the emphasis is on geometrical methods in Astronomy. The historical contents go back to Eratosthenes of Cyrene (about 275 – 195 B.C.) third librarian of the Alexandria Library, Aristarchus of Samos (about third century B.C.), Galileo Galilei (1564 – 1642), Halley (1656-1742).

The educational value/efficacy of the activity "Investigating the near Space by means of Geometry" (INV-SP-GEO) has been studied with a sample of forty-two students in the age range (16 -19).

This activity, developed in early 2012, proposes the reconstruction of several famous astronomical experiences involving geometrical Euclidean methods to estimate dimensions and distances in Solar System, e.g. Earth radius, angular dimension of Sun and Moon, Moon diameter, Moon and Sun distances to the Earth, size of Moon's mountains and distance of closer stars. The activity aims at helping teachers and students in studying and understanding some Astronomical contents of the Italian Secondary School scientific curricula (last three years, students' age about 16-19). The astronomical aspects and properties of the Solar System, star constellations and other celestial objects acquire tangible evidence in the dynamic representation on the dome, the learning processes are therefore facilitated. Crucial developments in astronomy and use of scientific method are addressed, together with their historical contexts and the links with conceptions and beliefs about Solar System and Universe.

The visualization from diverse reference systems (Earth, Moon, Sun, appropriate points in the Space) allows to correlate different viewpoints and to learn how to choose the most useful system according the observations to focus on and the results aimed at. Observations as well as measures are done by the participants; descriptive and interpretative models are discussed.

To frame historically the content of the activity it is useful to recall that Sun and Moon are the only two sky objects with finite dimensions at naked eye and that, around 2000 B.C., Egyptians and Babylonians had already estimated their angular dimension, from the Earth viewpoint, obtaining the same data (about 0,5°).

In the following the content of the activity is described.

Eratosthenes (around 240-230 B.C.) suggested a first geometrical method to estimate the Earth radius (Fischer, 1975). Accepting the hypothesis of a spherical Earth, he linked several pieces of astronomical knowledge experimentally based: 1) the Sun is very far from the Earth: from the geometrical viewpoint its light can be represented as parallel rays; 2) the length of shadow projected on the Earth surface by a vertical object (obelisk, column, stick, …) changes during the day, from sunrise to sunset; 3) each day, this shadow has a (daily) minimum, at 12 a.m. local time (noon); 4) each year the shadow daily minimum is minimum in the day of Summer Solstice (June 21st in the Earth Northern Hemisphere; 5) at Summer Solstice, 12 a.m local time, there are places on Earth surface where the vertical object projects no shadow. In the hypothesis of a spherical Earth, any vertical object (e.g. an obelisk) has the direction of the Earth radius.

Moreover, Eratosthenes knew that the Egyptian cities of Alexandria and Siene (now Aswan) had the same local time, namely they are on the same meridian, at a one degree approximation. Thus, in Alexandria and Siene, at Summer Solstice at 12 a.m., Sun light (i.e. rays in terms of mathematic model) illuminating an obelisk creates, respectively, a finite shadow and no shadow. Eratosthenes's bright idea was to measure the length of the shadow in Alexandria and to compute the angle between Sun rays and the obelisk (i.e. vertical direction). The (arc) distance between Alexandria and Siene was a well known value because of the caravanned path length journey estimated time). Due to the celebrated Euclid theorem (a straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the interior angles on the same side equal to two right angles, Euclid, The Elements, Book I, Proposition 29), the angle at the centre of the Earth is equal to the measured angle in Alexandria. Eratosthenes computed the Earth radius by the proportion: ?°: 360° = l: c, where ?° is the measured angle in Alexandria, l is the length of the arch between Alexandria and Siene and c is the length of the complete circle i.e. the local meridian (a meridian is the great circle passing through the geographic poles of the Earth and a specific location).

The method proposed by Aristarchus to estimate the radius and the distance of the Sun and Moon handles similar simple concepts of Euclidean 2D Geometry. An exhaustive and extensive description can be found in Gomez (2012). The PdC activity is then devoted to show the "renaissance" of those geometrical methods (applied in Astronomy since Egyptian and Roman ages) due to Galileo. Since the end of November 1609, Galileo studied the Moon with his 20x telescope[1] and published the results in *Sidereus Nuncius* (The Starry Messenger). He saw small dark spots, never seen before, on the illuminated part of the Moon's surface, and small light spots in the dark area (Figure 2).

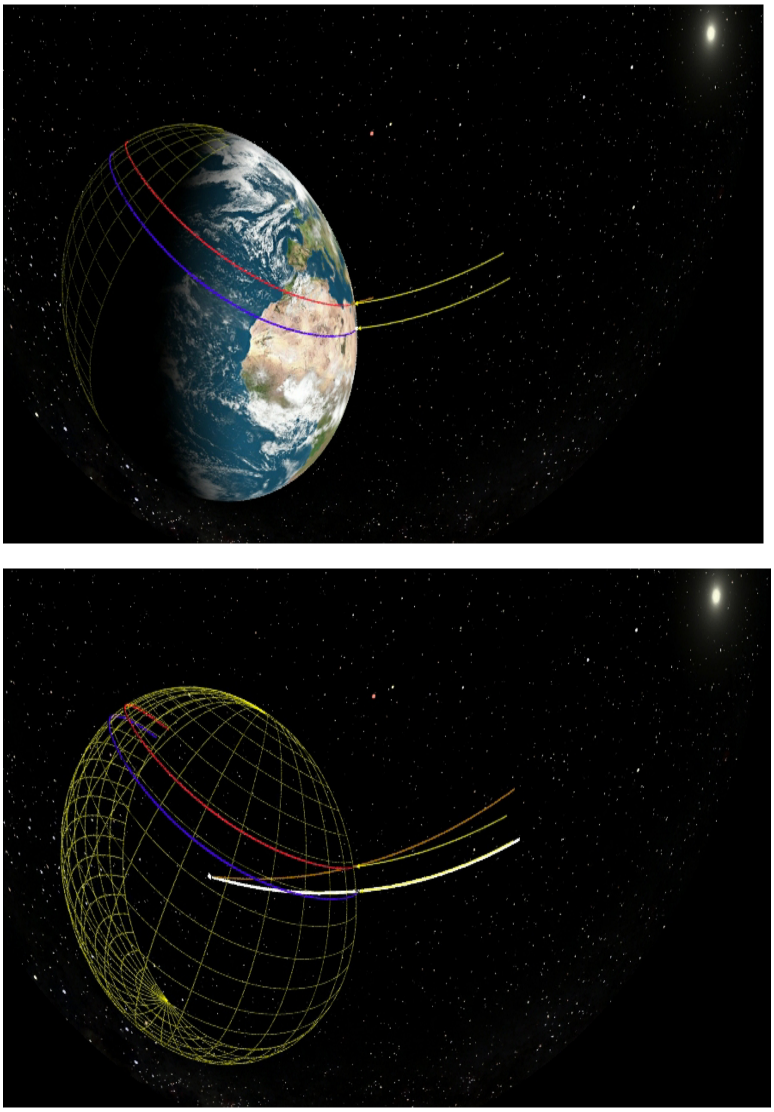
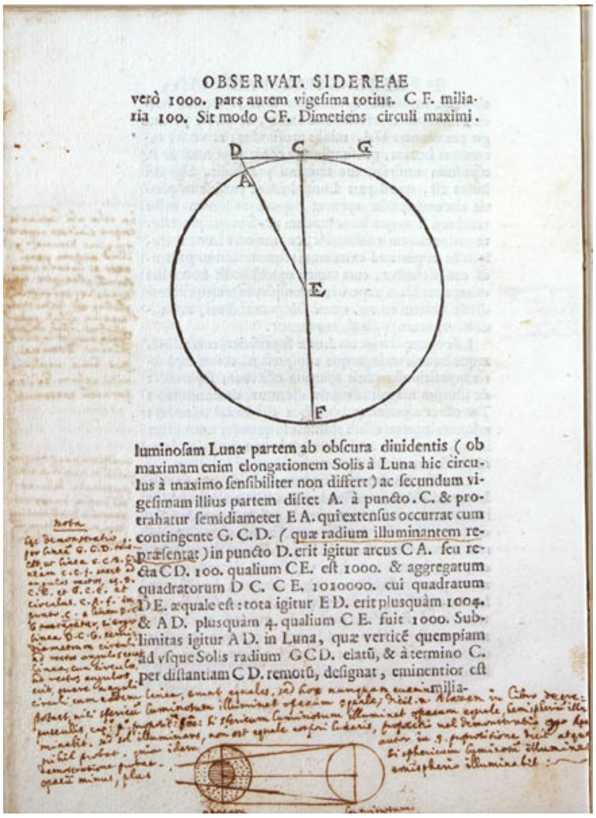


Figure 1. Eratosthenes method to estimate the Earth radius as seen in part of the planetarium dome. The realistic representation (top) and the schema (bottom). The Sun is the top-right bright spot. Due to the *full-dome* format (polar projection) to transfer the images from the dome spherical surface to a plane surface, there are several distortions: the solar *rays* do not appear straight.

As time passed, these spots varied, becoming lighter and eventually disappearing or becoming darker and more distinct. The spots "have a dark part on the side toward the Sun while on the side opposite the Sun

1 Galileo's first telescope was 3x, then he showed an 8x to the Doge in Venetia, later he worked with a 20x

they are crowned with brighter borders like shining ridges, as when a mountain is reached by Sun light before the valleys". The terminator, line between light and dark, was uneven; Moon was not a perfectly smooth sphere. Moon's surface has valleys, plains and mountains as the Earth. How can the moon, a heavenly body, not be perfect and spherical? If the Moon is imperfect, could there be other imperfect heavenly bodies as well? If heavenly bodies can be imperfect, why can the Earth not be a heavenly body? Measuring the distance of the bright spots from the terminator in units of Moons' radius, a simple application of Pythagoras theorem provides the height of the mountain.


OBSERVAT. SIDEREAE

verò 1000. pars autem vigesima totius. C F. milia-ria 100. Sit modo CF. Dimetiens circuli maximi.

luminosam Lunæ partem ab obscura diuidentis (ob maximam enim elongationem Solis à Luna hic circu-lus à maximo sensibiliter non differt) ac secundum vi-gesimam illius partem distet A. à puncto. C. & pro-trahatur semidiameter E A. qui extensus occurrat cum contingente G. C. D. (quæ radium illuminantem re-præsentat) in puncto D. erit igitur arcus C A. seu re-cta C D. 100. qualium C E. est 1000. & aggregatum quadratorum D C. C E. 1010000. cui quadratum D E. æquale est : tota igitur E D. erit plusquàm 1004. & A D. plusquàm 4. qualium C E. fuit 1000. Sub-limitas igitur A D. in Luna, quæ verticē quempiam ad vsque Solis radium G C D. elatū, & à termino C. per distantiam C D. remotū, designat, eminentior est milia-

Figure 2. A page of Sidereus Nuncius about the determination of Moon mountains height (left); the annotations in the margin should be by Galileo (Biblioteca Nazionale Centrale di Firenze). The original Galileo's autograph drawing of the observed effects (right).

The last interesting application of simple geometrical concept in Astronomy was provided by Edmond Halley (1716) in his famous proposal submitted to the Royal Society. He suggested to measure the solar parallax (and then the solar distance) observing the transit of Venus from two widely separated places. This has been the conjectural base for annual parallax of a star (near to the Earth) in order to measure the distance from Earth to the star (Hoskin, 1997). The first successful measurements of stellar parallax were made by Friedrich Georg Wilhelm von Struve in 1837 for the star Vega, shortly followed by Friedrich Bessel determination for the star 61 Cygni. In the INV-SP-GEO activity we propose the Sirius parallax and its distance determination.

Method

As other PdC activities, INV-SP-GEO proposes to teachers and students a full-dome real-time dynamic experience; a professional scientific tutor helps and the participants can freely interact with him/her.

The historical contexts, together with the astronomical properties of the sky bodies involved, are shown and discussed; the participants perform observations and the original measurements and compare them with the mathematical models; the geometrical proofs are proved step-by-step in a movie-like structure. Each astronomical experiment is reconstructed with a high verisimilitude with respect to the related natural phenomena, the participants' attention and motivation is increased by the immersion into a full-dome realistic experience.

Historical images (portrait of scientists, front cover of their original books, appropriate music, recorded narration selected from historical treatises and real-time talk of the tutor, etc.) evocate the cultural atmosphere of each astronomical/geometrical experiment and aim at increasing the emotional, attentive and cultural participation of the audience. The tangible evidence of the proposed contents helps the learning and teaching processes.

The educational side is reinforced also for the general public, to feed their basic science knowledge.

As first phase of the evaluation of the education value/efficacy of INV-SP-GEO the opinions of eighty teachers of Secondary school has been analysed, after they had participated to the activity. Both contents and structure of INV-SP-GEO have been positively evaluated. Later a test with a sample of students has been conducted with forty-two students (age 17-19) divided in two groups.

The students have been selected from two different Scientific Lyceums and a Technical Institute for Building and Surveying.

Twenty-two students are so-called "naïve or no-expert" (N) students; they have no particular interest for Mathematics or other Sciences and have not participated to Olympiads or special school programs on Mathematics, Astronomy or Physics. Twenty students are so-called "expert" (E) ones who have shown specific interest for Mathematics or other Sciences and have attended some of the above mentioned programs. Due to the difficulties to select an equilibrated sample with respect to male and female (in the Italian technical school there are always much more males than females), we have no statistic elements to take into account the gender effect in the results.

The students did not know the content of INV-SP-GEO. The activity has been held in the afternoon, after the regular school program, in the middle of the week, about at the end of the school year when written tasks, recitations and assessments are due. The educational value/efficacy of INV-SP-GEO has been tested in a diverse situation with respect to the above mentioned usual protocol for schools (3 phases: pre, at PdC, post). The aim was to see how the activity improves the students' knowledge even if they had not been previously exposed to the specific astronomical content via class-work, textbooks study, web-search, discussions with peers and teachers …). During INV-SP-GEO, the students have answered two questionnaires, pre and post activity, in about twenty minutes for each questionnaire.

After the activity, in the whole group discussion, the most of students has expressed appreciations and positive comments. Nevertheless, several students have explicitly highlighted their difficulties in handling mathematical contents (even if already well known by them) in astronomical topics that were unknown or insufficiently understood.

The pre-activity questionnaires aimed at knowing the students' knowledge on contents relevant to the INV-SP-GEO activity.

The post questionnaire aimed at testing the effects of the activity with respect to understanding the proposed topics and problems. The post questions are closely linked with the pre-activity ones.

The two questionnaires are reported in Appendix. The subjects of the questions are: n. 1- angular size of objects, n. 2- Earth radius estimate, n. 3 – stellar motions and parallax, n. 4 – Sun-Moon size and distance. Question n. 5, in pre- and post-questionnaires, tests the geometrical skills. In Table 1 the evaluation grid to rank the answers of each student is shown.

Table 1. Evaluation grid used in assessing the INV-SP-GEO activity.

Score	Description
0	No answer.
1	No significant knowledge.
2	The student perceives the physical problem or describes it but s/he does not answer correctly.
3	The student roughly understands the physical problem or describes it and s/he gives an answer close to the correct one.
4	The student understands the physical problem, describes it and gives a correct answer.
5	The student deeply understands the physical problem, describes it, gives the correct answer and proposes an appropriate formal / mathematical description of the phenomenon.

Data and Findings

The analysis of the collected questionnaires of two groups (N = Naïve; E = Expert) is based on the:

1. response scores of each group for each question, pre versus post test;
2. cognitive gain of each group for each question (as the % of the difference pre / post);
3. % of the total correct responses of the two groups for each question, pre / post;
4. overall gain of the two groups for each item (the % difference in input and output)
5. distribution of cognitive gain for each subject in group N and group E, for each question.

Here the main results.

Question 1: angular dimension

In group N, 28% of students improve their score, increasing the values of the response codes (1 - 3 positions). Nevertheless, the total amount of appropriate and correct answers decreases from 72% to 68%. In group E, 40% of students improve their score, 45% keeping the previous one. The correct answers increase from 60% to 80%. (Fig. 3).

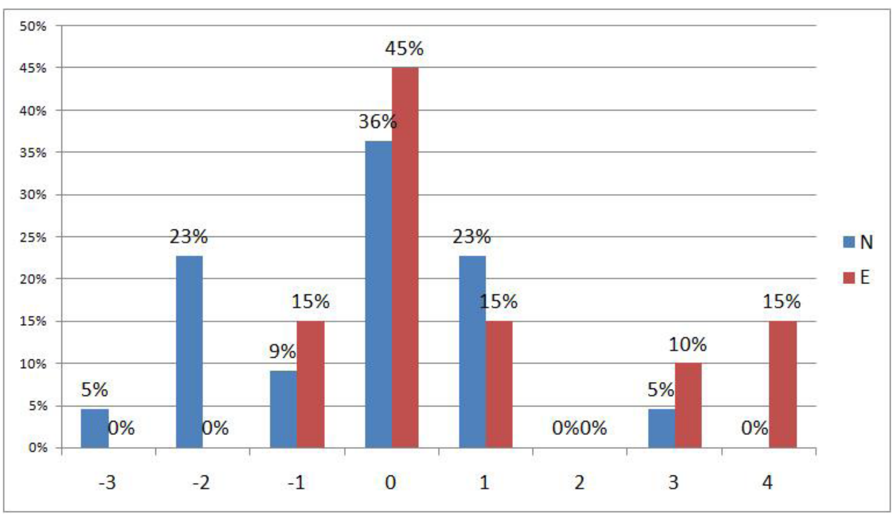


Figure 3. Question 1 about angular size. Difference, for each student, between the scores in pre- post-activity questionnaires. Experts (red), Naïve (blue).

Question 2: estimation of the Earth's radius

In group N, 46% of the students improve the score (increases by 1, 2 or 3 positions), the total correct answers move from 28% to 42%. In group E, 65% of students improve the score (up to 4 positions), the correct answers increase from 30% to 60%. (Fig. 4)

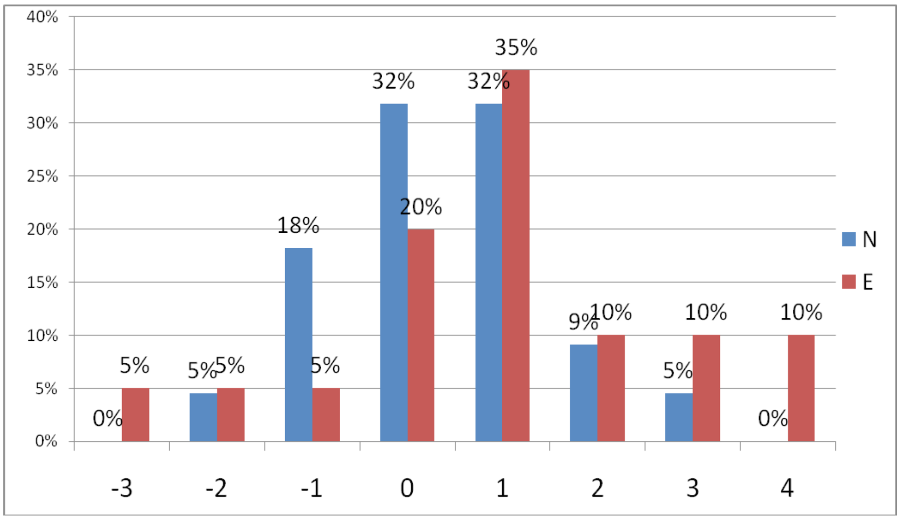


Figure 4. Question 2 about Earth radius estimate. Difference, for each student, between the scores in pre-post-activity questionnaires.

Question 3: star motion and parallax.

51% of the N group students increase the score from 1 to 4; the correct answers increase from 32% to 51%. In group E, 40% of students improve the score; 20% of them keep the previous score; the number of correct answers is stable (70%) (Fig. 5).

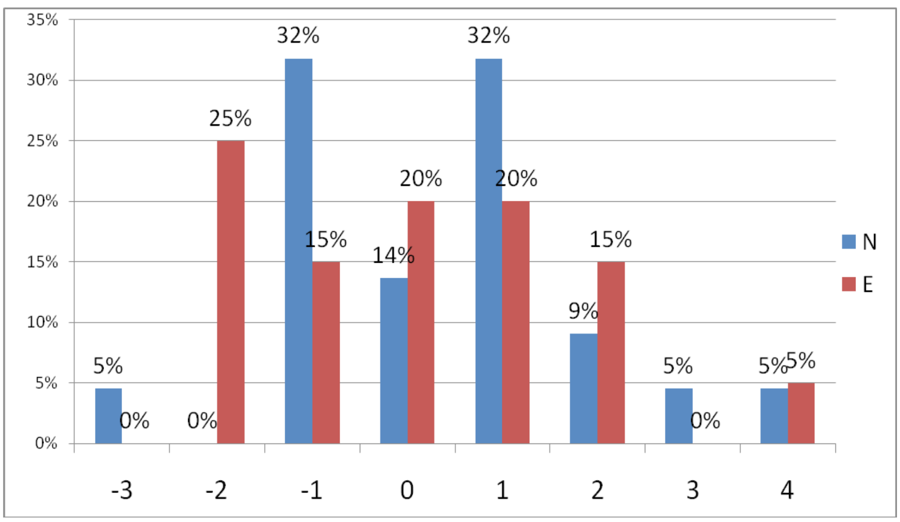


Figure 5. Question 3 about star motion and parallax Difference, for each student, between the scores in pre- post-activity questionnaires.

Question 4: distances Earth – Sun – Moon

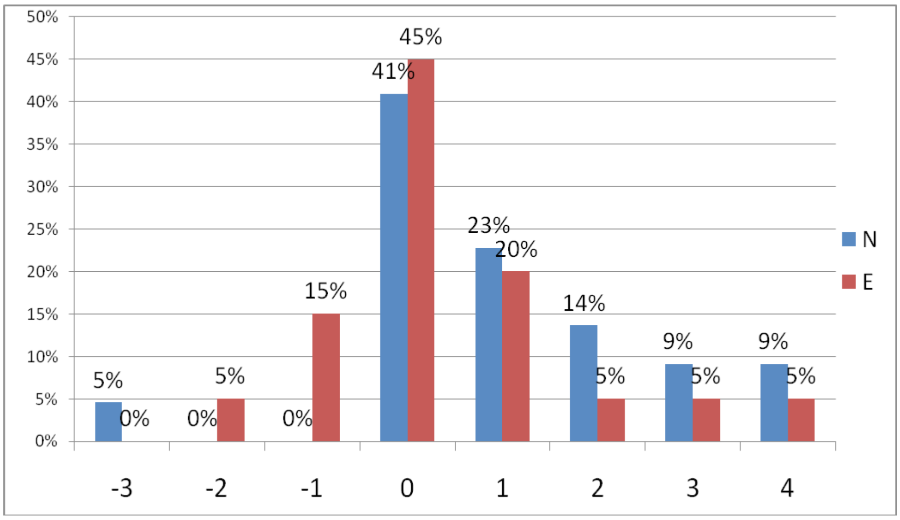


Figure 6. Question 4 about distances Earth – Sun – Moon, Difference, for each student, between the scores in pre- post-activity questionnaires.

In group N, 55% of the students improve the score (increases from 1 to 4 positions). In group E, 35% of students improve scores and 45% preserves the pre-questionnaire score. The percentage of correct answers is low: - pre-questionnaire, N = 5%; E = 35%; post-questionnaire, N = 19%; E = 45% (Fig.6).

Question 5: basic geometric background

Geometric basic knowledge differences between N and E group outgoes from the answers to Questions 5 (Fig. 6). The cumulative score of E group for appropriate abilities is 74%; the N one is 59%.

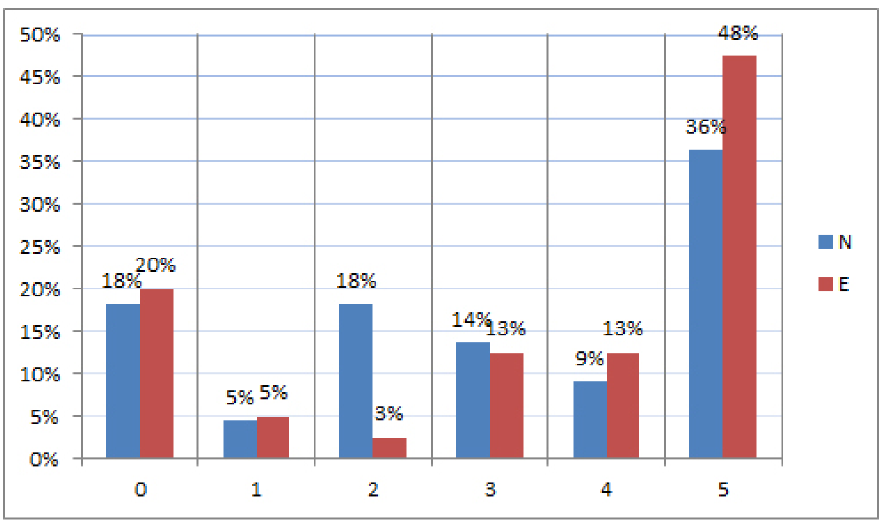


Figure 7. Question 5 on basic geometrical background. Cumulative scores in pre and post-questionnaire.

Table 2 shows the cumulative scores of good pre/post understanding astronomical topics (score ? 3), for the entire sample of students (N+E).

Table 2. Good pre/post understanding (score ? 3) of astronomical topics in INV-SP-GEO activity.

Topic	Pre	Post
Angular dimension	63%	75%
Earth radius estimate	31%	40%
Stars motion & parallax	50%	59%
Sun, Moon radius & distance	19%	41%

Discussion and Conclusions

Globally, the percentage of non-response decreases and that of correct answers increases. In general, the group E seems more prudent in giving answers than N group: the percentage of non-response is higher in pre-questionnaire. In particular, the E group students probably do not seem to be able to exploit in a balanced way the time available for the test; the last Question 5 is dealt with at the end of the allotted time in both pre and post questionnaires. In N group, instead, the no-response percentage increases, in particular for Question 5. This fact may likely be explained by a cognitive task perceived as too high (such to make the student tired or discouraged to answer) and/or by a greater awareness in trying to synthesize and use the contents, acquired in the activity, in order to provide an answer. In mean, except for Question 5, the non-response percentage decreases (mean values between E and N group and pre/post results: Q1: 15% / 0%; Q2: 15% / 11%; Q3: 14% / 11%; Q4: 3% / 3%; Q5: 17% / 21%).

Questions 1 (pre and post) are focused on angular dimension, a topic rarely taught in secondary school. In the best case, it is briefly addressed in Fine Arts, when discussing the consequences related to the Renaissance perspective, but with no mathematical approach. The E group seems to recognize the link between perspective and angular size, the correct responses increasing by 20%; the N group does not.

The best performances come from Questions 2 (pre and post) on the Earth radius estimate. The school programme introduces the Eratosthenes's method, and, probably, the students (more in E than in N) knew the content from a theoretical point of view. Then, the INV-SP-GEO allows them to gain a deep comprehension. This gain is less evident in N group.

The star motion and the parallax (Questions 3) appear to be known astronomical topics in the E group (high understanding rated to 70% both in pre and post). Likely, due to the dynamical tools used in INV-SP-GEO activity, the N group profits of this aspect and increases the response score from 32% to 51%.

The results of Questions 4 on Earth-Sun-Moon size and relative distances indicate that a high percentage of students show severe difficulties. While almost every student knows that the Moon is the satellite of the Earth, 60% of the E+N group do not extrapolate from this "dogmatic" knowledge the appropriate answer to the pre question. They are not able to transform their information into operational knowledge; the answers to the pre question are completely wrong or not significantly correct. Very likely the previous knowledge acquired at school is so "dogmatic", resistant, common and deeply rooted that 40% of students (either E or N group) achieves an output code 1 (No significant knowledge). Globally, the percentage of correct responses (score ? 3) increases, but the absolute percentage of E and N groups reveals low significant effect of INV-SP-GEO.

The results seem to validate the research hypotheses:

- the integration of Digital Planetarium (DP) activities in the syllabus of an ordinary secondary school is a reliable path;

• the contribution of a specific DP activity to the improvement of historical, astronomical and geometrical knowledge of secondary school students is positive.

As far as the EAV of INV-SP-GEO is concerned, both N and E groups provide similar global indications: the approaches of the activity encourage developments and improvements of the educational strategy, the specific astronomical contents and the historical framework.

The main useful didactical optimisations of the activity can be summarised as:

- to relax the density of the INV-SP-GEO contents showed in the 45' duration of the activity in PdC, to gain the participants experience a more comfortable pace;
- to address the main geometrical aspects during the class-work preparing the activity at the Planetarium;
- to include in the INV-SP-GEO storyboard a sort of "educative interface" between the usual Geometry studied in class and on textbooks and the Geometry applied to Astronomy and celestial space problems.

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Appendix

Pre-activity questionnaire to evaluate INV-SP-GEO impact

In your own words, answer the questions on the following five situations:

1. With only one eye, look at 1 euro coin located at 30 cm far from you; then at 60 cm. Describe which changes you observe:
2. Knowing the length of the Italian peninsula and having available an Earths' image, how do you measure the Earth radius?
3. Looking at the starry night sky on June 21st and December 21st, is the apparent motion of the stars the same? For all the stars? Do the relative positions change? And if so, how? If not, why?
4. Is it nearest the Moon to the Earth or the Earth to the Sun? Always? What is the astronomical phenomenon showing that it is not necessary to go into space and observe the three heavenly bodies to answer the question?
5. Complete the following theorems (the first one, attribute to Thales is known at least since the seventeenth century B.C. in Mesopotamia).

   A beam of parallel lines intersected by two transverses determines on them ....

   Two parallel lines intersected by a transverse form ....

Post-activity questionnaire to evaluate INV-SP-GEO impact

In your own words, answer the questions on the following five situations:

1. This is the "Scala Regia" in the Vatican built by Gian Lorenzo Bernini, in the years 1658-1661. Why do columns at rear appear shorter? Explain.

2. You cannot measure the shadow in Alexandria at noon on the Summer Solstice but you can only do it at noon of the Spring Equinox. To estimate the dimension of the Earths' radius, where should be measured the second shadow and what items do you need to know to perform the estimate?
3. The distance between the star Procyon and Earth is 4/3 of the distance of Earth/Sirius. Is the parallax of Sirius, with respect to Procyon one: larger / smaller / equal / not comparable (due to the different directions of two stars in space). Comparing photographs taken at six months, has moved more Sirius or Procyon with respect to distant stars? Why?
4. Why annular eclipses do also occurs (i.e. a solar eclipse in which the Moon covers all but a bright ring around the circumference of the Sun) in addition to the total solar eclipse?
5. Taken two triangles with angles ? = 30 ° and ? = 60 ° degrees; if the ratio between the lengths of the two smaller sides is 2, how much is the ratio between the two major sides? How does the relationship if the angles ? and? ? are all both identical in the two triangles?